Pair correlations in strongly coupled dusty plasmas

Xiaogang Wang and A. Bhattacharjee
Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52240
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Pair correlation functions and static structure factors for dust particles are calculated in a strongly coupled plasma using the Kirkwood approximation [J. Chem. Phys. **3**, 300 (1935); **14**, 180 (1946)]. The kinetic integral equation is solved in long- as well as short-wavelength regimes. In the long-wavelength regime, the pair correlation function exhibits Debye-Hückel behavior and decays exponentially with distance. This type of behavior occurs not only when the dust particles are weakly coupled but also when they are unannealed and in a moderate strongly coupled regime (characterized by dust coupling constants of order unity). In the short-wavelength regime, when the characteristic spatial scale is of the order of the interdust distance, the static structure factor for strongly coupled dust particles (characterized by coupling constants much larger than unity) exhibits pronounced oscillations and liquidlike behavior, as seen in recent experiments and particle-in-cell simulations. [S1063-651X(98)13710-8]

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I. INTRODUCTION

Dusty plasmas in the laboratory are typically three-component, quasineutral plasmas consisting of electrons, ions, and negatively charged dust particles. The coupling parameter for dust particles, which is defined as the ratio of the average (unshielded) Coulomb potential energy to the average kinetic energy, can be written as $\Gamma_d = Q_d^2/4\pi\epsilon_0 d_d T_d$, where Q_d is the dust charge, T_d is the dust temperature, n_d is the average dust density, and $d_d \equiv (3/4\pi n_d)^{1/3}$ is the average interdust distance (or the Wigner-Seitz radius). The dust particles, because of their large charge, are often in the strongly coupled regime, that is, $\Gamma_d \gg 1$, where they can become liquid-like and even freeze to form crystals [1–4], a behavior that has no counterpart in conventional, weakly coupled plasmas.

The nature of particle correlation functions in strongly coupled dusty plasmas is largely an unexplored subject, although a few analytical results have appeared recently [5–7]. Fortunately, the matter has not been left entirely in the realm of analytical theory. There now exist experimental measurements [8,9] of the pair correlation function in a strongly coupled dusty plasma that pose a challenge for theory. Furthermore, there are particle-in-cell (PIC) simulation results [10] that suggest that dust pair correlations are qualitatively different in an unannealed, moderately strongly coupled ($\Gamma_d \gtrsim 1$) dusty plasma supporting long-wavelength fluctuations than in an annealed, strongly coupled dusty plasma ($\Gamma_d \gtrsim 1$) in which these fluctuations are removed by damping. Whereas unannealed dust particles show a persistence of Debye-Hückel behavior with correlations decaying exponentially with distance, annealed dust particles exhibit long-range correlations typical of a liquid or a crystal.

Several features of the pair correlation function seen in experiments or simulations of strongly coupled dusty plasmas are similar to those seen in liquids or one-component plasmas. (See, for instance, [11] and references therein.) However, the theoretical explanation of these experimental or numerical observations requires more than mere rederivation of results known from the theory of liquids. One of the outstanding difficulties of dusty plasma physics is that the average interaction potential between two dust particles is not known precisely either experimentally or theoretically. A related difficulty is that the shielding properties of dust particles are not sufficiently well understood. It is commonly assumed [12] that the average interaction potential between two dust particles separated by a distance r is of the Debye-Hückel or Yukawa type, that is, given by

$$\Phi(r) = Q_d^2 \exp(-r/\lambda_{dp})/4\pi\epsilon_0 r, \tag{1}$$

where $\lambda_{Dp} = (\lambda_{De}^{-2} + \lambda_{Di}^{-2})^{-1/2}$ and $\lambda_{De,i} = (\epsilon_0 T_{e,i}/n_{e,i}e^2)$. Note that the characteristic shielding distance λ_{Dp} has no dependence on the dust species. This would appear to be a reasonable assumption if the dust particle density is sufficiently low. However, in many dusty plasmas it is fairly common to have many electrons, ions, and dust particle in an electron or ion Debye sphere. In such cases it is clear that, on relaxation time scales short compared to the dust plasma time scale ($\sim 2\pi\omega_{pd}^{-1}$, where ω_{pd} is the dusty plasma frequency), the charges behave as in a conventional plasma in which Debye shielding will extend to a distance λ_{De} on the electron plasma time scale ($\sim 2\pi\omega_{pe}^{-1}$) or λ_{Dp} on the ion ($\sim 2\pi\omega_{pi}^{-1}$) time scale. It is not obvious, however, what the shielding length should be on relaxation time scales on the order of, or much longer than, the dust plasma time scale, which is long enough for the dust particles to move and participate in the shielding process. It was proposed in [5] that on these rather long time scales, if the plasma is not annealed or crystallized but gaslike, the correct shielding distance for point dust particles should be $\lambda_D = (\lambda_{Dp}^{-2} + \lambda_{Dd}^{-2})^{-1/2}$, where $\lambda_{Dd} = (\epsilon_0 T_d/n_d Q_d^2)^{1/2}$ is the dust Debye length. The one-dimensional PIC simulations [10] tend to support this conclusion in the moderate strong-coupling regime ($\Gamma_d \gtrsim 1$) when the interdust particle distance is of the order of λ_{Dd}. However, when the dust particles are annealed by frictional damping so that the kinetic energy of the dust particles is reduced and the strong-coupling regime ($\Gamma_d \gg 1$) becomes accessible, the simulations show liquidlike

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correlations that are qualitatively different from the gaslike Debye-Hückel correlations seen in the moderate strong-coupling regime.

A theory of correlation functions in strongly coupled dusty plasmas must account for measurements such as those reported in [8] and [9] as well as the range of behavior seen in the PIC simulations [10]. In this paper we present a kinetic theory of pair correlation functions that makes progress towards meeting that challenge. We do so by truncating the exact Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy according to the Kirkwood approximation [13]. This method of truncation yields an integral equation for the pair correlation function, discussed in Sec. II. Solutions of the Kirkwood equation are presented and compared with experimental results in Sec. III. In the long-wavelength regime, when the discreteness of dust grains can be neglected because the wavelengths are much larger than the interdust distance, we recover the results obtained from an earlier kinetic theory [5] as well as hydrodynamics [7]. However, when the model is extended to include the short-wavelength regime in which the wavelength is of the order of the interdust spacing, we find static structure factors (or pair correlations) qualitatively similar to those reported in [8–10].

II. THE KIRKWOOD APPROXIMATION

In order to keep this discussion self-contained, we begin with a brief review of the relevant equations of the BBGKY hierarchy and the Kirkwood approximation [11,13,14]. The exact electrostatic Klimontovich-Dupree equation is

$$\left[\frac{\partial}{\partial t}+L_\alpha(\mathbf{X})-\sum_\beta \int d\mathbf{X}'\, V_{\alpha\beta}(\mathbf{X},\mathbf{X}')N_\beta(\mathbf{X}')\right]N_\alpha(\mathbf{X})=0, \tag{2}$$

where $N_\alpha=\sum_{i=1}^{N_{\alpha 0}}\delta(\mathbf{X}-\mathbf{X}_i)$ is the microscopic distribution function of $N_{\alpha 0}$ point particles of type α, each with charge Q_α and mass m_α, $\mathbf{X}\equiv(\mathbf{x},\mathbf{v})$ is a point in μ space, $L_\alpha(\mathbf{X})\equiv\mathbf{v}\cdot\boldsymbol{\nabla}$,

$$V_{\alpha\beta}(\mathbf{X},\mathbf{X}')\equiv\frac{Q_\alpha Q_\beta}{m_\alpha}\frac{\partial}{\partial\mathbf{x}}\,\phi(\mathbf{x}-\mathbf{x}')\cdot\frac{\partial}{\partial\mathbf{v}}, \tag{3}$$

and $\phi(\mathbf{x}-\mathbf{x}')=(4\pi\epsilon_0|\mathbf{x}-\mathbf{x}'|)^{-1}$ is the Coulomb interaction potential per unit positive charge. The first three multiparticle distribution functions are

$$F_\alpha(1)=\langle N_\alpha(1,t)\rangle, \tag{4a}$$

$$F_{\alpha\beta}(12)=\langle N_\alpha(1,t)N_\beta(2,t)\rangle-\delta_{\alpha\beta}\delta(1-2)F_\alpha(1), \tag{4b}$$

$$\begin{aligned}F_{\alpha\beta\gamma}(123)=&\langle N_\alpha(1,t)N_\beta(2,t)N_\gamma(3,t)\rangle\\&-\delta_{\alpha\beta}\delta_{\beta\gamma}\delta(1-2)\,\delta(2-3)F_\alpha(1)\\&-\delta_{\alpha\beta}\delta(1-2)F_{\beta\gamma}(23)\\&-\delta_{\beta\gamma}\delta(2-3)F_{\gamma\alpha}(31)\\&-\delta_{\gamma\alpha}\delta(3-1)F_{\alpha\beta}(12),\end{aligned} \tag{4c}$$

where $i\equiv\mathbf{X}_i$ and $\langle\ \rangle$ denotes an average. The first two equations of the BBGKY hierarchy are

$$\left[\frac{\partial}{\partial t}+L_\alpha(1)\right]F_\alpha(1)=\sum_\beta\int d2\; V_{\alpha\beta}(12)F_{\alpha\beta}(12), \tag{5a}$$

$$\begin{aligned}&\left[\frac{\partial}{\partial t}+L_\alpha(1)+L_\beta(2)-[V_{\alpha\beta}(12)+V_{\beta\alpha}(21)]\right]F_{\alpha\beta}(12)\\&\quad=\sum_\gamma\int d3[V_{\alpha\gamma}(13)+V_{\beta\gamma}(23)]F_{\alpha\beta\gamma}(123).\end{aligned} \tag{5b}$$

The multiparticle correlation functions $G_{\alpha\beta},H_{\alpha\beta\gamma}$ are defined by the relations

$$F_{\alpha\beta}(12)=F_\alpha(1)F_\beta(2)+G_{\alpha\beta}(12), \tag{6a}$$

$$\begin{aligned}F_{\alpha\beta\gamma}(123)=&F_\alpha(1)F_\beta(2)F_\gamma(3)+F_\alpha(1)G_{\beta\gamma}(23)\\&+F_\beta(2)G_{\gamma\alpha}(31)+F_\gamma(3)G_{\alpha\beta}(12)\\&+H_{\alpha\beta\gamma}(123).\end{aligned} \tag{6b}$$

Substituting Eqs. (6) in Eqs. (5), it follows that

$$\left[\frac{\partial}{\partial t}+L_\alpha(1)-\sum_\beta\int d2\; W_{\alpha\beta}(12)F_\beta(2)\right]F_\alpha(1)=0, \tag{7a}$$

$$\begin{aligned}&\left[\frac{\partial}{\partial t}+L_\alpha(1)+L_\beta(2)\right]G_{\alpha\beta}(12)\\&\quad-\sum_\beta\,[W_{\alpha\beta}(12)+W_{\beta\alpha}(21)]F_\alpha(1)F_\beta(2)\\&\quad=\sum_\gamma\int d3[V_{\alpha\gamma}(13)F_\alpha(1)G_{\beta\gamma}(23)\\&\quad+V_{\beta\gamma}(23)F_\beta(2)G_{\gamma\alpha}(31)]\\&\quad+\sum_\gamma\int d3[V_{\alpha\gamma}(13)+V_{\beta\gamma}(23)]\\&\quad\times[F_\gamma(3)G_{\alpha\beta}(12)+H_{\alpha\beta\gamma}(123)],\end{aligned} \tag{7b}$$

where

$$W_{\alpha\beta}(12)\equiv V_{\alpha\beta}(12)\left[1+\frac{G_{\alpha\beta}(12)}{F_\alpha(1)F_\beta(2)}\right]. \tag{8}$$

Equations (7) are the first two of an infinite hierarchy of equations and an approximation needs to be introduced at this stage to truncate the hierarchy. Numerous sophisticated approximation schemes, such as the hypernetted chain approximation, are known from the theory of liquids [11] but require extensive numerical work. We demonstrate below that the simpler Kirkwood approximation [11,13,14] does remarkably well in accounting for salient features of the observations in real [8,9] as well as numerical [10] dusty plasma experiments. Following Kirkwood [13], we write

$$H_{\alpha\beta\gamma}(123)=\frac{G_{\alpha\beta}(12)G_{\beta\gamma}(23)}{F_{\beta}(2)}+\frac{G_{\beta\gamma}(23)G_{\gamma\alpha}(31)}{F_{\gamma}(3)}+\frac{G_{\gamma\alpha}(31)G_{\alpha\beta}(12)}{F_{\alpha}(1)}+\frac{G_{\alpha\beta}(12)G_{\beta\gamma}(23)G_{\gamma\alpha}(31)}{F_{\alpha}(1)F_{\beta}(2)F_{\gamma}(3)}. \quad (9)$$

This approximation enables a truncation of the hierarchy and yields the following closed equations for F_α and $G_{\alpha\beta}$:

$$\left[\frac{\partial}{\partial t}+L_{\alpha}(1)-\sum_{\beta}\int d2\, W_{\alpha\beta}(12)F_{\beta}(2)\right]F_{\alpha}(1)=0, \quad (10a)$$

$$\left[\frac{\partial}{\partial t}+L_{\alpha}(1)+L_{\beta}(2)\right]G_{\alpha\beta}(12)-\sum_{\beta}[W_{\alpha\beta}(12)+W_{\beta\alpha}(21)]F_{\alpha}(1)F_{\beta}(2)$$
$$=\sum_{\gamma}\int d3\, W_{\alpha\gamma}(13)\left(F_{\alpha}(1)G_{\beta\gamma}(23)+F_{\gamma}(3)G_{\alpha\beta}(12)+\frac{G_{\alpha\beta}(12)G_{\beta\gamma}(23)}{F_{\beta}(2)}\right)$$
$$+\sum_{\gamma}\int d3\, W_{\beta\gamma}(23)\left(F_{\beta}(2)G_{\gamma\alpha}(31)+F_{\gamma}(3)G_{\alpha\beta}(12)+\frac{G_{\gamma\alpha}(31)G_{\alpha\beta}(12)}{F_{\alpha}(1)}\right). \quad (10b)$$

For a homogeneous and isotropic plasma in thermal equilibrium, we have

$$F_{\alpha}(1)=F_{M\alpha}(\mathbf{v}_1)f_{\alpha}(\mathbf{x}_1)=F_{M\alpha}(\mathbf{v}_1)n_{\alpha}, \quad (11)$$

where $F_M(\mathbf{v})$ is the Maxwellian distribution, $n_\alpha=N_{\alpha 0}/V$ is the average number density of point particles of type α, and V is the plasma volume. Under these conditions, we can write

$$G_{\alpha\beta}(12)=F_{\alpha}(1)F_{\beta}(2)g_{\alpha\beta}(r_{12}), \quad (12)$$

where $g_{\alpha\beta}(r_{12})\equiv g_{\alpha\beta}(|\mathbf{x}_1-\mathbf{x}_2|)$ is the pair correlation function. It follows that

$$W_{\alpha\beta}(12)=-\mathbf{v}_1\cdot\frac{\partial}{\partial \mathbf{x}_1}\psi_{\alpha\beta}(r_{12})\equiv-\frac{Q_{\alpha}Q_{\beta}}{T_{\alpha}}[1+g_{\alpha\beta}(r_{12})]\mathbf{v}_1\cdot\frac{\partial\phi(r_{12})}{\partial \mathbf{x}_1}. \quad (13)$$

Then Eqs. (10a) and (10b) reduce, respectively, to

$$\mathbf{v}_1\cdot\frac{\partial}{\partial \mathbf{x}_1}\left[1+\sum_{\beta}\int d\mathbf{x}_2 f_{\beta}(\mathbf{x}_2)\psi_{\alpha\beta}(r_{12})\right]f_{\alpha}(\mathbf{x}_1)=0 \quad (14a)$$

and

$$(\mathbf{v}_1-\mathbf{v}_2)\cdot\frac{\partial}{\partial \mathbf{x}_1}[g_{\alpha\beta}(r_{12})+\psi_{\alpha\beta}(r_{12})]$$
$$=-\mathbf{v}_1\cdot\frac{\partial}{\partial \mathbf{x}_1}\sum_{\gamma}\int d\mathbf{x}_3 f_{\gamma}(\mathbf{x}_3)\psi_{\gamma\alpha}(r_{31})\times[g_{\alpha\beta}(r_{12})+g_{\beta\gamma}(r_{23})+g_{\alpha\beta}(r_{12})g_{\beta\gamma}(r_{23})]$$
$$-\mathbf{v}_2\cdot\frac{\partial}{\partial \mathbf{x}_2}\sum_{\gamma}\int d\mathbf{x}_3 f_{\gamma}(\mathbf{x}_3)\psi_{\beta\gamma}(r_{23})\times[g_{\gamma\alpha}(r_{31})+g_{\alpha\beta}(r_{12})+g_{\gamma\alpha}(r_{31})g_{\alpha\beta}(r_{12})]. \quad (14b)$$

For a homogeneous system, the integro-differential equation (14b) can be solved by means of Fourier transforms. We introduce the dimensionless Fourier pair correlation function $c_{\alpha\beta}(\mathbf{k})\equiv n_\alpha g_{\alpha\beta}(\mathbf{k})$ and interaction function $\pi_{\alpha\beta}(\mathbf{k})\equiv n_\alpha\psi_{\alpha\beta}(\mathbf{k})$. The Fourier transform of Eq. (14b) yields

$$\mathbf{k}\cdot(\mathbf{v}_1-\mathbf{v}_2)[c_{\alpha\beta}(\mathbf{k})+\pi_{\alpha\beta}(\mathbf{k})]+\mathbf{k}\cdot\sum_{\gamma}\times[\mathbf{v}_1\pi_{\alpha\gamma}(\mathbf{k})c_{\gamma\beta}(\mathbf{k})-\mathbf{v}_2\pi_{\beta\gamma}(-\mathbf{k})c_{\gamma\alpha}(-\mathbf{k})n_\alpha/n_\beta]$$
$$+\frac{\mathbf{k}}{n_\beta}\cdot\sum_{\gamma}\int d\mathbf{p}[\mathbf{v}_1\pi_{\alpha\gamma}(\mathbf{k}-\mathbf{p})c_{\gamma\beta}(\mathbf{k}-\mathbf{p})c_{\beta\alpha}(\mathbf{p})-\mathbf{v}_2\pi_{\beta\gamma}(\mathbf{p}-\mathbf{k})c_{\gamma\alpha}(\mathbf{p}-\mathbf{k})c_{\alpha\beta}(-\mathbf{p})]=0, \quad (15)$$

where

$$\pi_{\alpha\beta}(\mathbf{k})=\frac{Q_\beta}{Q_\alpha}\frac{k_{D\alpha}^2}{k^2}\left[1+\frac{1}{n_\alpha}\int d\mathbf{p}\,\frac{\mathbf{k}\cdot\mathbf{p}}{p^2}\,c_{\alpha\beta}(\mathbf{k}-\mathbf{p})\right]. \quad (16)$$

Note that Eq. (16) follows by taking the Fourier transform of Eq. (13) and substituting $\phi_{\alpha\beta}(\mathbf{k})=(\epsilon_0 k^2)^{-1}$ for Coulomb interactions.

III. SOLUTIONS OF THE KIRKWOOD EQUATION

We consider first a conventional electron-ion plasma without dust particles, with the Debye length given by $\lambda_{Dp}=(\lambda_{De}^{-2}+\lambda_{Di}^{-2})^{-1/2}$, where $\lambda_{De,i}\equiv(\epsilon_0 T_{e,i}/n_{e,i}e^2)^{1/2}$. We scale the wave number by the Debye length λ_{Dp} and rewrite the integrals over $\mathbf{p}$ in Eqs. (15) and (16) as

$$\frac{1}{n_0}\int d\mathbf{p}\xrightarrow[\mathbf{p}\to\mathbf{p}/\lambda_{Dp}]{}\frac{1}{n_0\lambda_{Dp}^3}\int d\mathbf{p}, \quad (17)$$

where $n_i=n_e=n_0$. For weakly coupled plasmas, the plasma parameter $g_p\equiv(n_0\lambda_{Dp}^3)^{-1}\ll 1$ and Eq. (15) reduces to an algebraic equation. It can then be shown that

$$c_{ee}(\mathbf{k})=c_{ii}(\mathbf{k})=-c_{ei}(\mathbf{k})=-c_{ie}(\mathbf{k})=-\frac{1}{2}\frac{k_{Dp}^2}{k^2+k_{Dp}^2}, \quad (18)$$

where $k_{Dp}=(\lambda_{De}^{-2}+\lambda_{Di}^{-2})^{1/2}\equiv(k_{De}^2+k_{Di}^2)^{1/2}$. The Fourier inversion of Eq. (18) yields the well-known Debye-Hückel correlation function

$$g_{\alpha\beta}(r)=\frac{Q_\alpha Q_\beta}{r}\,e^{-k_{Dp}r}, \tag{19}$$

where $\alpha,\beta=e,i$.

When dust particles are introduced in a plasma, the density of dust particles plays a crucial role in determining the shielding and correlation properties of the plasma. If the density of dust particles is so low that there is less than one dust particle in the plasma Debye sphere, i.e., $d_d \gg \lambda_{Dp}$, then the relevant shielding distance for a dust particle is λ_{Dp}. However, in a moderate strong-coupling regime, it is fairly common to have many electrons, ions, and dust particles in the plasma Debye sphere. Then the dust particles within a plasma Debye sphere ''see'' each other and it is not unnatural to expect the shielding distance to be different from the plasma Debye length λ_{Dp}.

It was proposed in [5] that if point dust particles are in a moderate strong-coupling regime where they are unannealed and gaslike, then the correlation function should be of the Debye-Hückel type with the screening length $\lambda_D=(\lambda_{Dp}^{-2}+\lambda_{Dd}^{-2})^{-1/2}$, where $\lambda_{Dd}\equiv(\epsilon_0 T_d/n_d Z_d^2 e^2)^{1/2}$ and $Z_d e$ is the charge on a dust particle. It follows that if $\lambda_{Dd}\ll\lambda_{Dp}$, then $\lambda_D \sim \lambda_{Dd}$ [5]. A similar conclusion also emerges from hydrodynamic theory, which is valid in the strong-coupling regime when the perturbations have low frequencies and long wavelengths [7]. We now deduce this result from the Kirkwood equation (15) in the long-wavelength regime.

We assume, as in [7], that the dust-dust correlation $c_{dd}(\mathbf{k})$ dominates over the correlation of dust with ions and electrons. Then Eq. (15) reduces to

$$c_{dd}(\mathbf{k})+\pi_{dd}(\mathbf{k})+\pi_{dd}(\mathbf{k})c_{dd}(\mathbf{k}) + \frac{1}{n_{d0}}\int d\mathbf{p}\;\pi_{dd}(\mathbf{k}-\mathbf{p})c_{dd}(\mathbf{k}-\mathbf{p})c_{dd}(\mathbf{p})=0. \tag{20}$$

Since the dust parameter $g_d\equiv(n_{0d}\lambda_{Dd}^3)^{-1}\equiv 4\sqrt{3}\pi\Gamma_d^{3/2}$, a large coupling parameter Γ_d for dust particles necessarily implies a large value for g_d. One cannot, under these conditions, make the weak-coupling approximation ($\Gamma_d\ll 1$), which leads to expressions such as Eq. (18) for conventional plasmas. However, in the long-wavelength regime we can scale the wave number by a typical length scale λ much larger than the interdust distance d_d, so that

$$\frac{1}{n_{0d}}\int d\mathbf{p} \xrightarrow[\mathbf{p}\to\mathbf{p}/\lambda]{} \frac{1}{n_{0d}\lambda^3}\int d\mathbf{p}. \tag{21}$$

Since $d_d\ll\lambda$, we have $n_{0d}\lambda^3\gg 1$. In this limit, all integral terms in Eqs. (15) and (16) can be ignored and we obtain

$$c_{dd}(\mathbf{k})+\pi_{dd}(\mathbf{k})+\pi_{dd}(\mathbf{k})c_{dd}(\mathbf{k}) \approx c_{dd}(\mathbf{k})+\frac{k_{Dd}^2}{k^2}+\frac{k_{Dd}^2}{k^2}\,c_{dd}(\mathbf{k})=0 \tag{22}$$

or

$$c_{dd}(\mathbf{k})\approx-\frac{k_{Dd}^2}{k^2+k_{Dd}^2}. \tag{23}$$

Equation (23) can be inverted to give

$$g_{dd}(r)\approx\frac{Z_d^2e^2}{T_d r}\,e^{-k_{Dd}r}, \tag{24}$$

which is similar to the result obtained from earlier kinetic theory [5]. Furthermore, from Eq. (23) we obtain the static structure factor

$$S_{dd}(\mathbf{k})=c_{dd}(\mathbf{k})+1\approx\frac{k^2}{k^2+k_{Dd}^2}, \tag{25}$$

which is identical to the result obtained in [7] from generalized hydrodynamics. In this limit, which is relevant when long-wavelength disturbances (such as dust-acoustic waves) are present in an unannealed gaslike plasma and dust particle discreteness is smoothed over, a strongly coupled dusty plasma behaves as if it were a Vlasov fluid. The pair correlation function shows the persistence of Debye-Hückel behavior with a screening distance determined by the dust Debye length, as seen in the unannealed PIC simulations [10].

An experimental test of the theoretically predicted role of the dust Debye length in the moderate strong-coupling regime $\Gamma_d \gtrsim 1$ appears difficult for a number of reasons. Note that when $\Gamma_d\gtrsim 1$, the dust Debye length is essentially of the same order as the interdust distance. The dust particles in present-day strong-coupling experiments are often large, with radii larger than the dust Debye length. If λ_{Dd} is smaller than the typical radius of dust particles, it is clearly not valid in the present context to assume, as we have, that the dust particles can be treated as point particles. In order to determine correlation functions that decay on the spatial scale of the dust Debye length, it would be necessary to work not only with dust particles of smaller radius but to use diagnostic techniques that can measure particle correlations on such short spatial scales. It would also be necessary to carry out the measurements in the moderate strong-coupling, gaslike regime $\Gamma_d\gtrsim 1$ where the excitation of significant fluctuations makes measurements difficult [15].

We now discuss solutions of the full Kirkwood integral equation, including the short-wavelength regime that recognizes dust particle discreteness. In order to include the short-wavelength regime, we scale the wave number by a typical length scale λ equal to the interdust distance d_d and write

$$\frac{1}{n_{0d}}\int d\mathbf{p} \xrightarrow[\mathbf{p}\to\mathbf{p}/\lambda]{} \frac{1}{n_{0d}\lambda^3}\int d\mathbf{p} = \frac{4\pi}{3}\left(\frac{d_d}{\lambda}\right)^3\int d\mathbf{p}=\frac{4\pi}{3}\int d\mathbf{p}. \tag{26}$$

Clearly, we must now retain every term in the Kirkwood equation (20) and the solution is expected to deviate significantly from that in the long-wavelength regime. The static structure factor can be written as

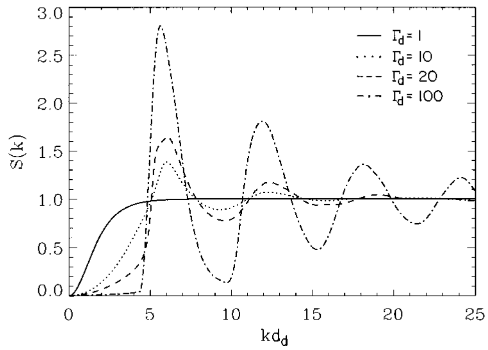

FIG. 1. Static structure factors for a dust particle, calculated from the Kirkwood equation (27), as a function of kd_d for various values of Γ_d: $\Gamma_d = 1$ (solid line), 10 (dotted line), 20 (dashed line), and 100 (dot-dashed line).

$$S_{dd}(\mathbf{k}) \equiv c_{dd}(\mathbf{k}) + 1 = \frac{k^2}{k^2 + k_D^2[1 + w(\mathbf{k})]}, \qquad (27)$$

where

$$w(\mathbf{k}) \equiv \frac{4\pi}{3} \int d\mathbf{p}\, \frac{\mathbf{k}\cdot\mathbf{p}}{p^2} S_{dd}(\mathbf{p})[S_{dd}(\mathbf{k}-\mathbf{p}) - 1]. \qquad (28)$$

We have solved the integral equation (27) numerically by iteration, assuming that the system is isotropic in k space, that is, $S_{dd}(\mathbf{k}) = S_{dd}(k)$. In Fig. 1 we plot the numerical solution for the static structure factor $S(k) \equiv S_{dd}(k)$ for $\Gamma_d = 1,10,20,100$. It is clear by inspection that the static structure factor is monotonic and qualitatively of the Debye-Hückel type when $\Gamma_d \sim 1$. However, when $\Gamma_d \sim 10$ nonmonotonic features appear as gentle (but significant) bumps that become more pronounced as Γ_d increases. When $\Gamma_d \sim 100$, the structure factor exhibits pronounced oscillatory features characterizing a liquid state.

In Fig. 2 we reproduce, for ease of reference, the experimental data on the static structure factor reported in [9] during a melting transition of a dusty plasma from a solid to a liquid phase, followed by a gaslike phase. During this transition, brought about by a gradual reduction of the gas pressure, the measured values of the coupling parameter Γ_d are as follows: $\Gamma_d = 3$ (at 39 Pa), 5 (at 42 Pa), 8 (at 45 Pa), 22 (at 55 Pa), 50 (at 67 Pa), and 100 (at 76 Pa). The theoretically calculated static structure factor shown in Fig. 1 is quantitatively similar to that shown in Fig. 2. The degree of agreement appears to be surprisingly good in view of the number of strong assumptions made in the theoretical model. For instance, the annealed experiment realizes a two-dimensional lattice and is anisotropic, whereas the theory assumes isotropy. In the experiment there are significant flows and wake effects [16] not included in the theory. The theory assumes that the dust particles are point objects with fixed charge. This assumption is certainly violated in the experiment where the dust particles have significant extension in space and the charge on the dust particles often varies as a function of time.

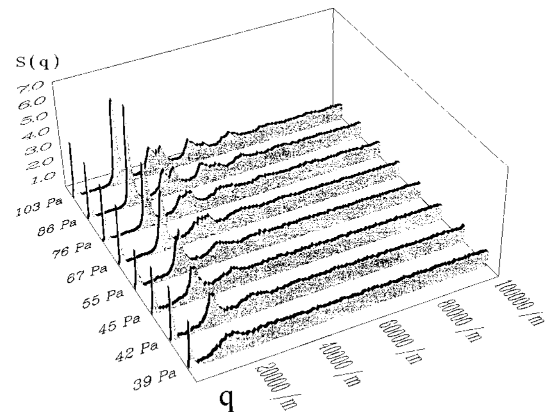

FIG. 2. Evolution of the static structure factor $S(q)$ of dust particles as a function of the wave number q in the experiment by Melzer *et al.* [9] for different values of the gas pressure (courtesy of Melzer).

The theoretically calculated pair correlations are also in accord with the PIC simulations presented in [10]. We have remarked earlier on the persistence of Debye-Hückel behavior for $\Gamma_d \sim 1$ in the unannealed simulation. While [5] and [7] identify correctly the occurrence of this type of behavior in the moderate strong-coupling, gaslike regime, they overestimate the range of Γ_d over which such behavior persists. In the annealed (or damped) simulations, significant deviations from Debye-Hückel behavior are seen in the range $\Gamma_d \sim 10-100$. The correlations are seen to be liquidlike when $\Gamma_d > 10$, which is consistent with the numerical solutions discussed above.

IV. SUMMARY

We have developed a kinetic description of pair correlations in strongly coupled dusty plasmas based on the Kirkwood approximation. The kinetic integral equation for the pair correlation function is solved analytically as well as numerically. In the long-wavelength regime, it is demonstrated that the dust correlation function exhibits exponential decay with distance characteristic of the Debye-Hückel solution. In this hydrodynamic regime, the discreteness of the dust particles is suppressed because the wavelength of the perturbations is much larger than the interdust distance. This regime is expected to persist in the moderate strong-coupling regime $\Gamma_d \gtrsim 1$ in an unannealed, gaslike dusty plasma.

When we extend the model to include the short-wavelength regime, where wavelengths are of the order of the interdust distance, liquidlike correlations emerge from the theory. The static structure factor, shown in Fig. 1, exhibits oscillatory features characteristic of strongly coupled systems. The predictions of theory compare favorably with experimental [8,9] and PIC simulation results [10].

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